A STUDY ON COMPLEX DEMODULATION OF THE GEOMAGNETIC TOTAL INTENSITY DATA OF TAIWAN

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台灣地磁全磁力資料之複數解調研究

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ABSTRACT

Complex demodulation (CMD), by the use of frequency shifting of the Fourier Transform, can be applied to filter out unitary frequency component from a time series. It can also be used to examine the time variations of the amplitude and the phase of selected frequency components of a time series. After a general review of the amplitude spectrum of the geomagnetic total intensity records at Lunping (LP) and Tsengwen (TW), it is found that there apparently exists a periodic component with a period of about 24 hours. The simple difference in geomagnetic total intensity between LP and TW also shows the congenial periodic phenomenon. Complex demodulation is applied to analyze the data of LP and TW for the purpose of examining this periodic phenomenon. We have computed 11 demodulates of the monthly series of hourly mean value at LP and TW, taking 1 cycle/day as the frequency component. The modulus and phase of these demodulates are obtained. The averages of moduli of these 11 series at LP and TW are 9.6 gamma and 13.2 gamma, respectively.

INTRODUCTION

By the advent of the FFT algorithm, various techniques of time series analysis have been developed by workers in many fields. Complex demodulation is one of these developments. Some tedious computations in time domain can be simplified by complex demodulation (CMD) method which peforms the operations in frequency domain.

The application of CMD was first introduced into the field of geophysics by Banks. The present paper, which describe the CMD technique and its computational implementation, relies heavily on that work. In this study, we have designed a computer program for the CMD method based on the formulations by Banks. The geomagnetic total intensity data recorded at Lunping (LP) and Tsengwen (TW) stations in northern and southern Taiwan (Figure 1), respectively, are processed by this program.

DEFINITION OF COMPLEX DEMODULATES

The process of complex demodulation as applied to a time series X(t) (assumed to be sampled at a series of points spaced at equal intervals of time (Δt) is defined in terms of two very simple mathematical operations. First, each frequency band of interest in the spectrum is shifted to zero frequency by multiplying each term of the time series by a complex exponential function $\exp(-i\omega' t)$, where ω' is the central frequency of the shifted band. A new series $Xs(\omega', t)$ is produced for each frequency band, i.e.

$$Xs(\omega', t) = x(t)\exp(-i\omega' t)$$

Second, the frequency-shifted series is then low-pass filtered using a set of weights a_k ($k = -m$ to $+m$, m = positive integer), and the result is the (complex) demodulated time series $Xd(\omega', t)$.

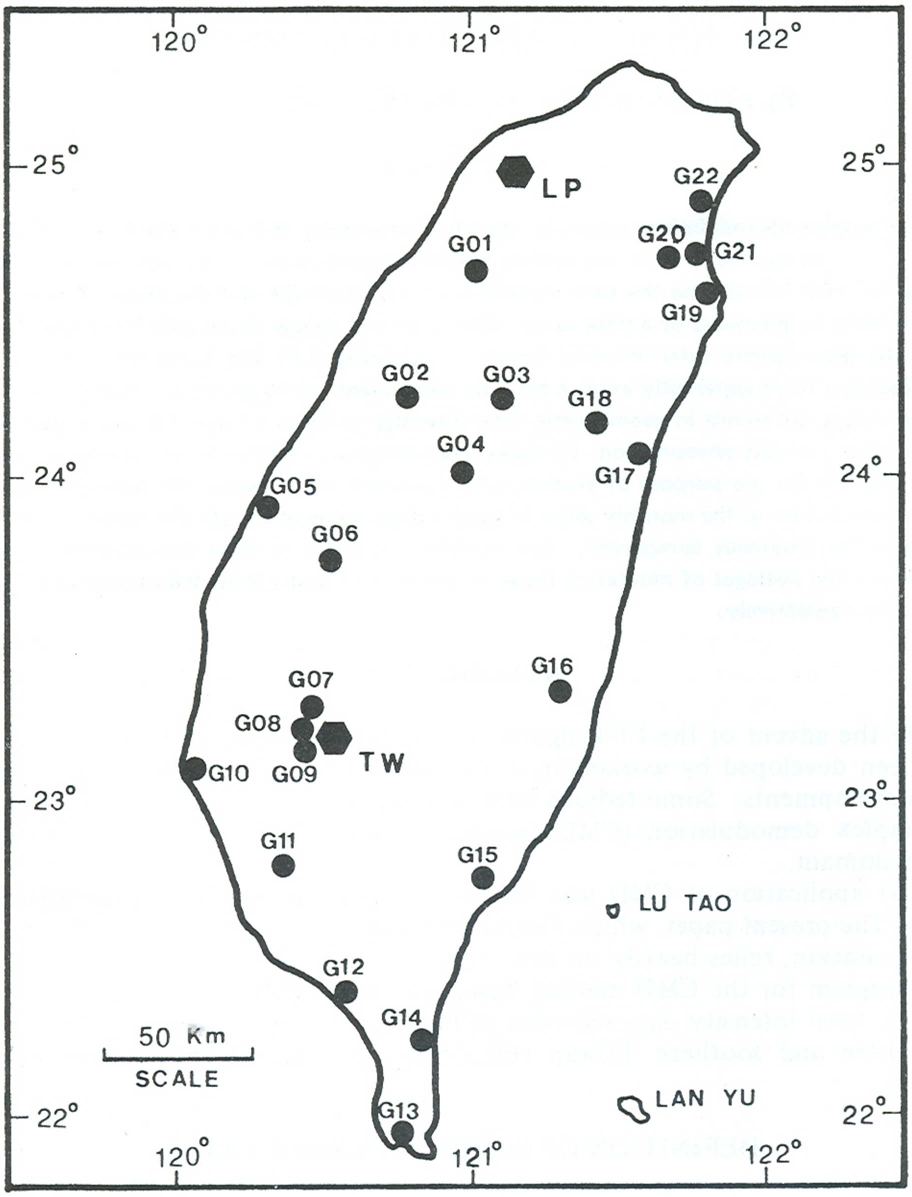

Figure 1. Network for observation of the geomagnetic total intensity in Taiwan. LP and TW are base stations. The others are repeat stations.

$$Xd(\omega',t) = \sum_{k=-m}^{k=+m} a_k Xs(\omega', t + k\Delta t)$$

The demodulates can be expressed most conveniently in the form

$$Xd(\omega',t) = |Xd(\omega',t)| \exp(-i\varphi(\omega',t))$$

in terms of the modulus and phase of $Xd(\omega',t)$.

DEMODULATION OF DATA CONTAINING PERIODIC COMPONENTS

Let us suppose that the data contain a periodic component that would produce a peak in its Fourier spectrum at frequency ω_0, i.e.

$$x(t) = A\cos(\omega_0 t + \gamma)$$

The simple complex demodulate centered on frequency $\omega' = \omega_0 + \delta\omega$ will be

$$\begin{aligned} Xs(\omega',t) &= (A/2)\exp(-i(\omega_0+\delta\omega)t)\{\exp i(\omega_0 t+\gamma)+\exp(-i(\omega_0 t+\gamma))\} \\ &= (A/2)\{\exp(-i(\delta\omega t-\gamma))+\exp(-i(2\omega_0+\delta\omega)t+\gamma))\} \end{aligned}$$

which contains frequencies $-\delta\omega$ and $-(2\omega_0+\delta\omega)$

The frequency-shifted series is then low-pass filtered, so hopefully the component at $-(2\omega_0+\delta\omega)$ is removed completely, leaving

$$Xd(\omega_0+\delta\omega,t) = (A/2)\exp(-i(\delta\omega t-\gamma))$$

Here we have assumed that the filter has unity response and introduces no phase shift at frequency $-\delta\omega$.

If we know that the data contain a periodic variation with a frequency of ω_0, it is natural to choose $\omega' = \omega_0$ as the central frequency of the demodulate, and to shift down to zero the band of frequencies immediately around ω_0. In that case, $\delta\omega = 0$, and

$$Xd(\omega_0,t) = (A/2)\exp i\gamma$$

i.e. the modulus of the demodulate is A/2 and its phase is equal to the phase of the periodic variation at ω_0. If A and γ change with time, there will be a corresponding change in the modulus and phase of the demodulate.

COMPUTATIONAL PROCEDURE

The direct procedure is to use the defining formulae, i.e., first to multiply each term of x(t) by $\exp(-i\omega' t)$, and then to low-pass filter the resultant data series. However, such a procedure is computationally expensive and leads to redundancy of information in the demodulated series.

Alternatively, a faster way of computing the demodulates is by means of the Fast Fourier Transform.

The raw data series is prepared for the FFT by the usual way of removing its mean and linear trend, tapering the ends to zero by half cosine bells, and padding zeros to bring the number of data points up to 2^k (where k is an integer). We then compute the FFT of the modified time series. The real and imaginary parts of the Fourier Transform are multiplied by a suitable discrete function of frequency centered on ω'. This function defines the filter which smoothes the series of raw demodulates Xs. The resultant band of frequencies is shifted to zero, care being taken to arrange the resulting negative frequency components in the correct storage locations. Finally, an inverse FFT operation converts the transforms back into the time domain, and generates the series of complex demodulates centered on frequency ω'.

The complete procedure is illustrated schematically in Figure 2. In the example shown, the spectrum of the data series has a peak at $\omega' = \omega_0$, and the demodulated band is naturally centered on $\omega' = \omega_0$.

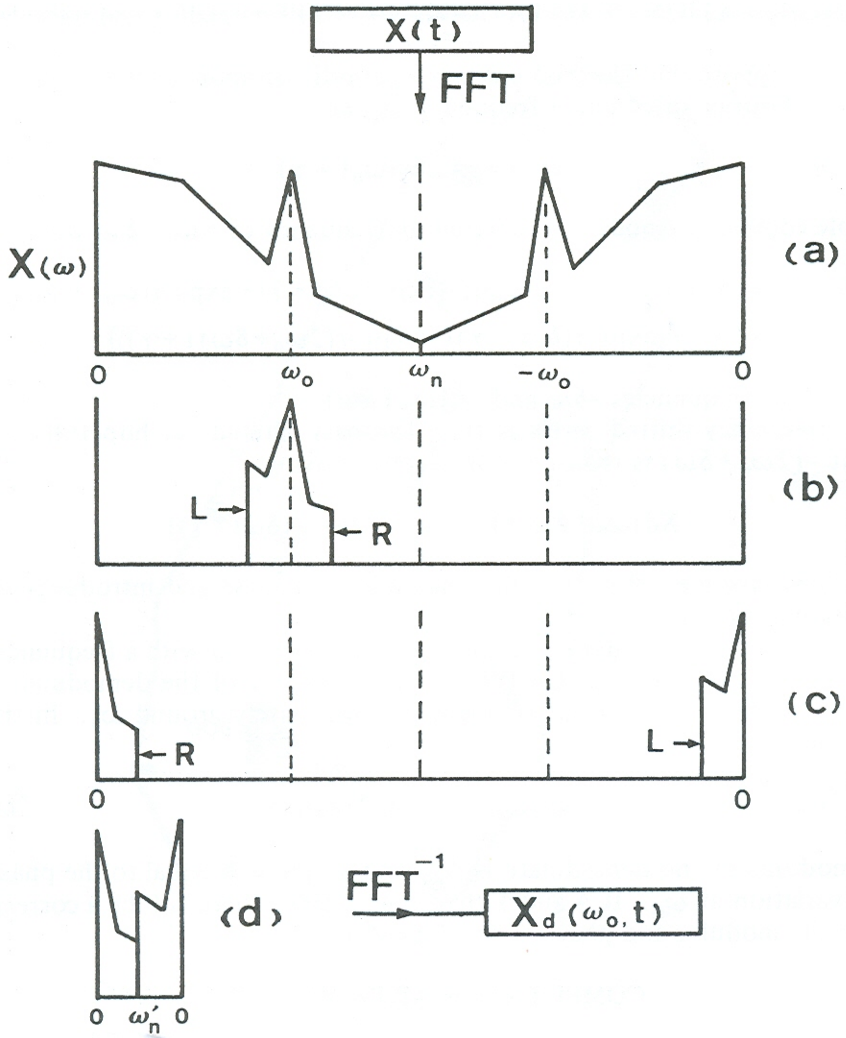

Figure 2. Schematic diagram of the procedure for computing complex demodulates Xd (ω_0, t) from the time series x(t). (a) Raw Fourier spectrum; (b) Filtered Fourier spectrum; (c) Frequency shifted Spectrum; (d) Truncated frequency shifted Spectrum. Note that no attempt has been made to indicate a realistic shape for the filter. (After Banks, 1975)

The low-pass filter used in this study is a consine bell, i.e.

$$W(\omega) = 1/2(1+\mathrm{COS}(2\pi(\omega-\omega')/\Delta\omega) \qquad \omega'-\Delta\omega/2 \leqslant \omega \leqslant \omega'+\Delta\omega/2$$
$$= 0 \qquad \text{elsewhere}$$

COMPLEX DEMODULATION OF SYNTHETIC WAVELETS

In order to check our CMD computer program, we use synthetic wavelets consisting of simple cosine waves and process them with this program. The results of these tests are satisfactory. One example of these tests is illustrated below. Let us suppose that S(t) contains three periodic components that would produce three peaks in its Fourier spectrum at frequency ω_1, ω_2 and ω_3. i.e.

$$S(t) = P(t) + Q(t) + R(t)$$

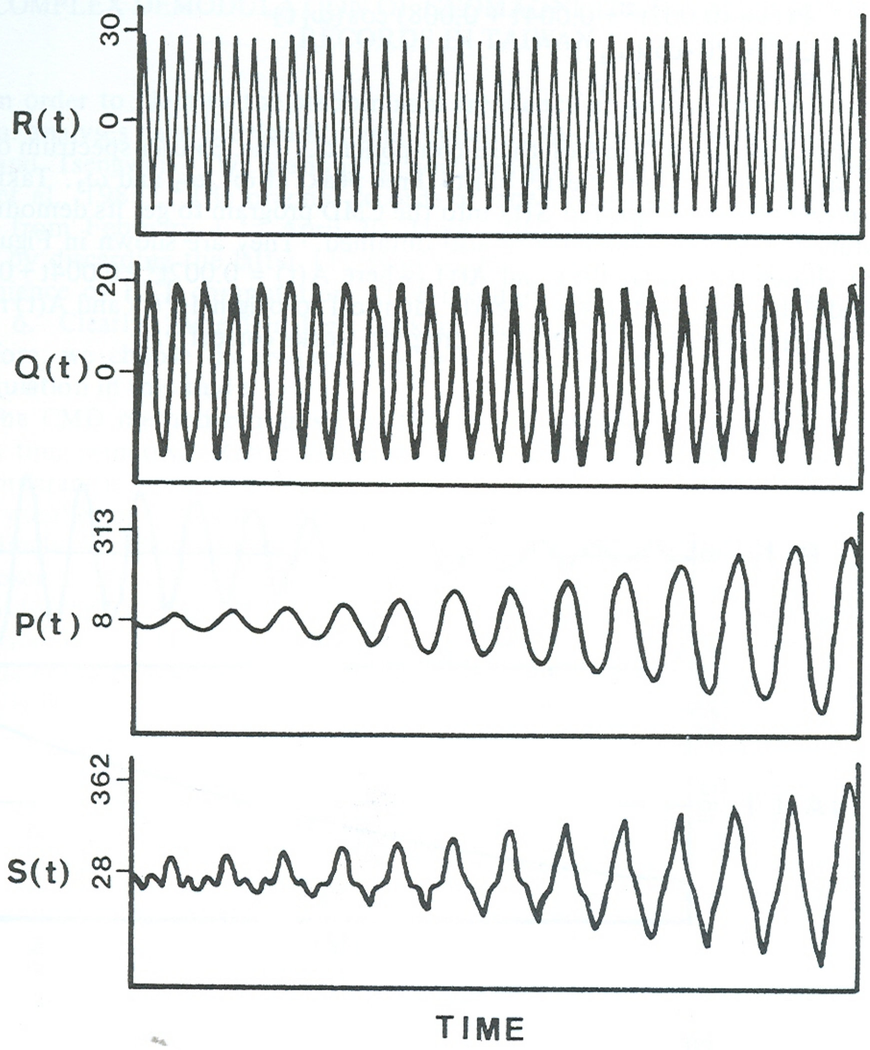

Figure 3. Wave form of three time series of cosine type and their sum at the bottom.

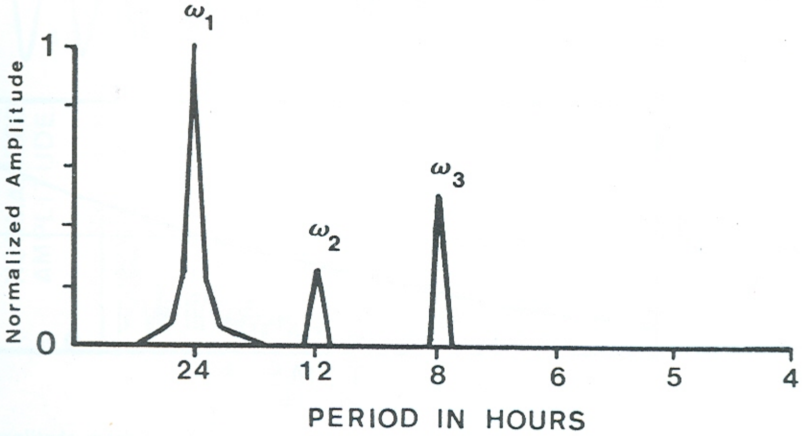

Figure 4. Amplitude Spectrum of S(t) in Figure 3.

where $P(t) = (0.002t^2 + 0.004t + 0.008)\cos(\omega_1 t)$
$Q(t) = 20 \cdot \cos(\omega_2 t)$
$R(t) = 30 \cdot \cos(\omega_3 t)$

Figure 3 shows the waveforms of these components. The Fourier spectrum of S(t) is shown in Figure 4. There clearly exists three peaks at ω_1, ω_2 and ω_3. Taking ω_1 as the central frequency, we fed S(t) into the CMD program to get its demodulates. The modulus of this demodulates is also obtained. They are shown in Figure 5b. Figure 5a shows the initial P(t) and A(t) (where $A(t) = 0.002t^2 + 0.004t + 0.008$). The demodulates and modulus are nearly identical to original P(t) and A(t) respectively. This demonstrates that the CMD program works correctly.

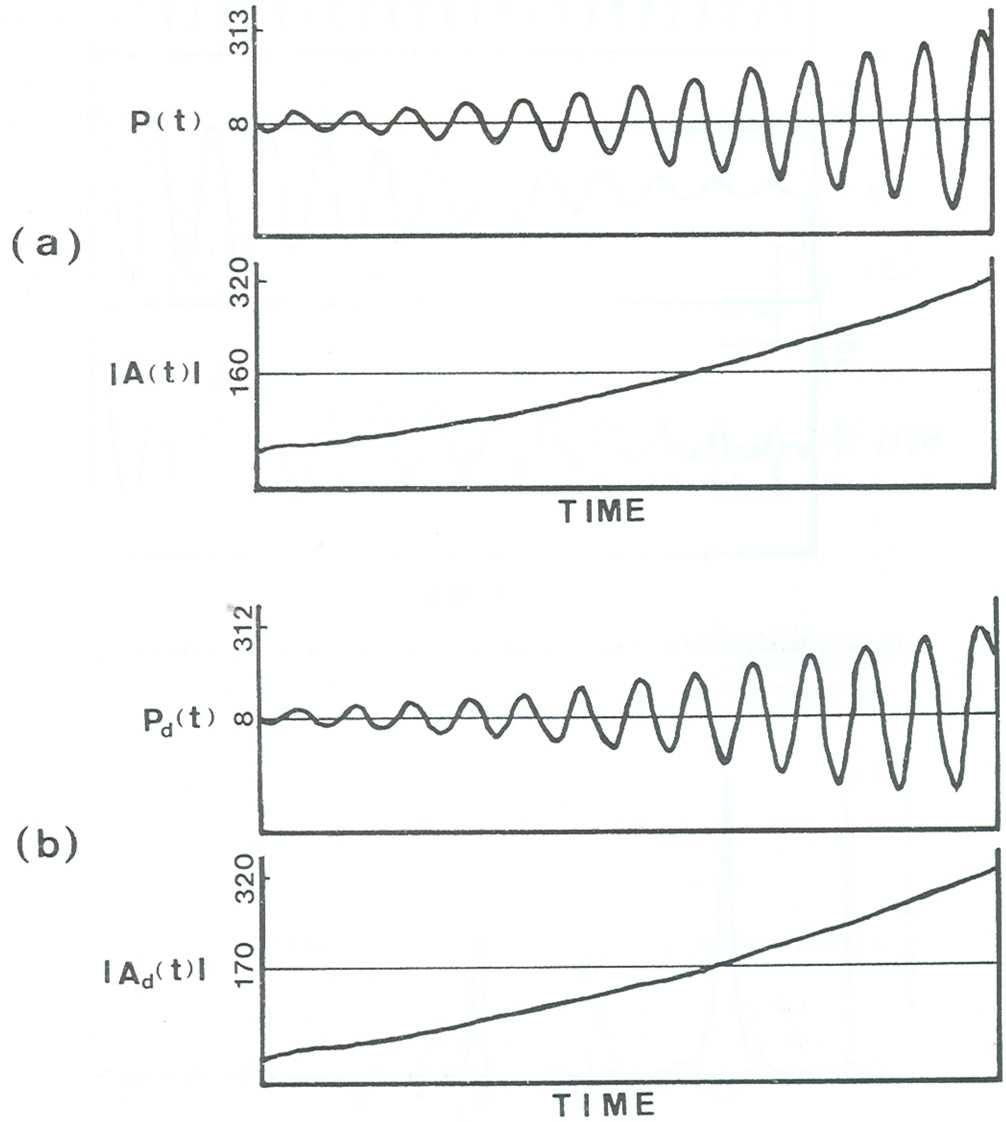

Figure 5. (a) Top: Original P(t) as a component of S(t). Bottom: A(t).
(b) Top: Calculated demodulates of S(t) at ω_1. Bottom: The corresponding modulus.

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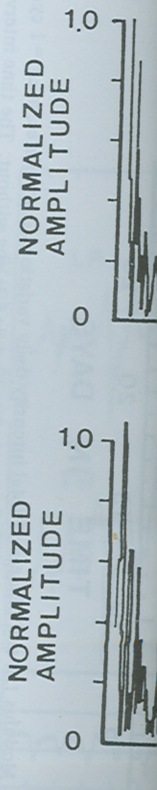

Figure 6. Amplitude spectrum
(top) and TW (bott

COMPLEX DEMODULATION OF GEOMAGNETIC TOTAL INTENSITY RECORDS IN TAIWAN

In order to identify the predominant frequency of geomagnetic total intensity, spectral analyses have previously been applied to the continuous records at Lunping (LP) and Tsengwen (TW) stations in northern and southern Taiwan, respectively (Yeh et al., 1981). The original records at one sample per hour have 2160 data points from February 1 to April 30, 1984. Its length has been reduced to 2048 points by discarding the latter 112 points (after removal of the mean, etc.) for the convenience of FFT computation. The resultant amplitude spectra are shown in Figure 6. Clearly, there is a high peak in the vicinity of the period of 24 hours. Therefore, we choose the frequency $\omega' = 1$ cycle/day as the central frequency for demodulation in this study.

The CMD method requires that the time series should be continuous over a chosen time window. Result achieved by putting the whole time series into the CMD program is approximately similar to that obtained by dividing the time series into a number of segments, analyzing each segment and combining them together. Accordingly, eleven monthly data sets from November, 1980 to November, 1984 are chosen to be processed by the CMD program because they are uninterrupted. The moduli and phases of these demodulated time series are calculated and a typical result is shown in Fig. 7 for the month of November 1984. The statistic mean of modulus of each month is shown in Figure 8. For each month, the value at TW station is always greater than that at LP station. Obviously, these two curves vary in

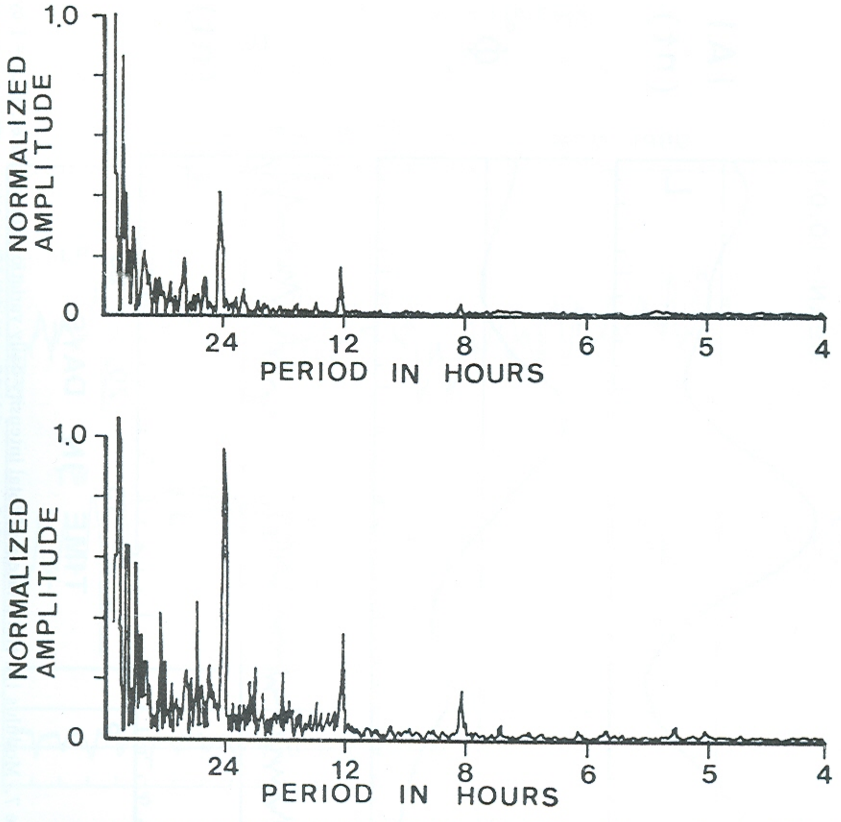

Figure 6. Amplitude spectrum of geomagnetic total intensity records from February 1 to April 30, 1984 at LP (top) and TW (bottom).

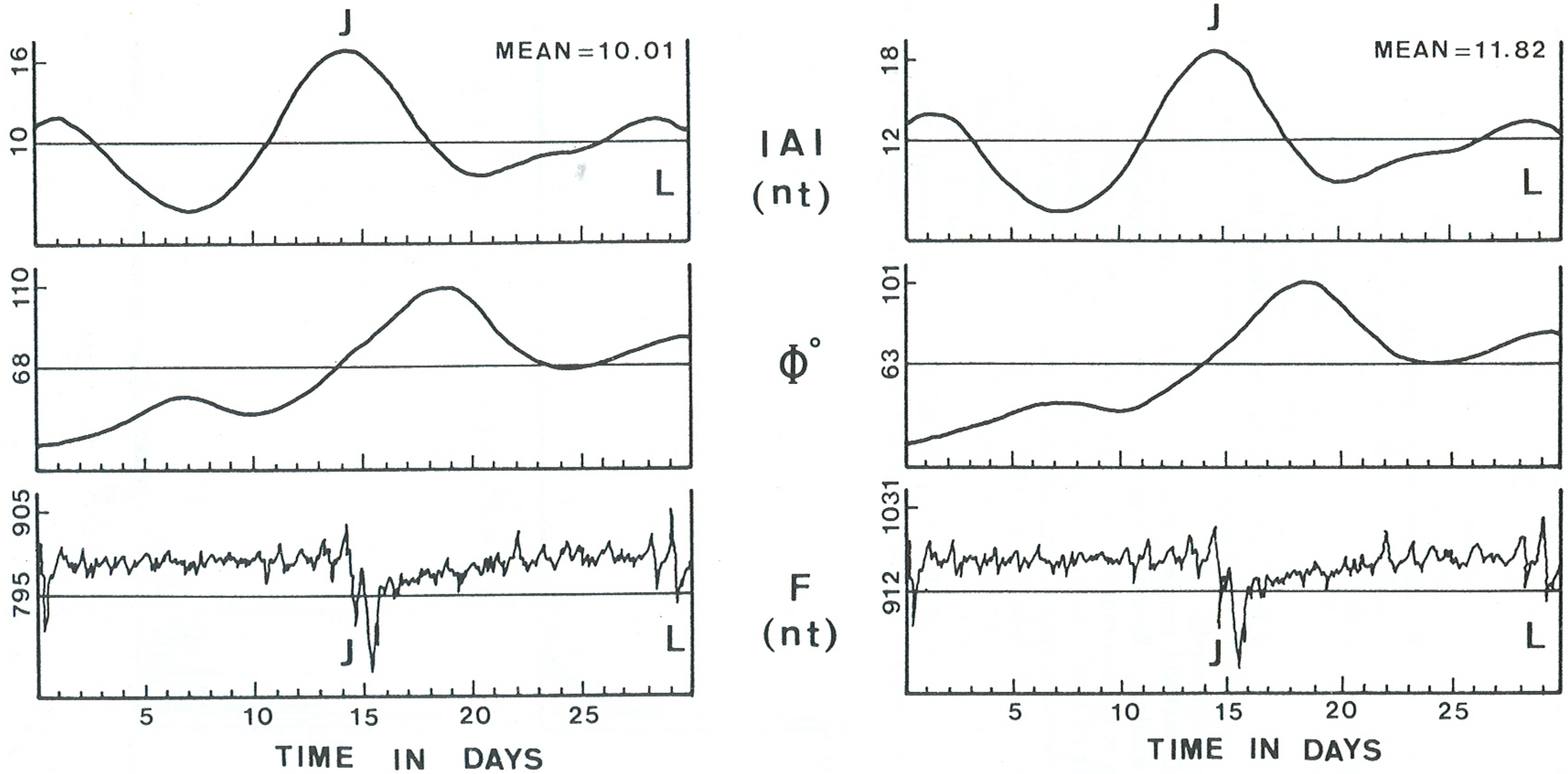

Figure 7. Modulus |A| and phase ϕ of total intensity daily variation demodulates, at $\omega' = 1$ cycle/day, and the corresponding magnetograms for November, 1984. J is a hump in |A(t)| with a corresponding magnetic storm while L is one without. The time interval between J and L is ~13.5 day which is typical for the |A(t)| curve.

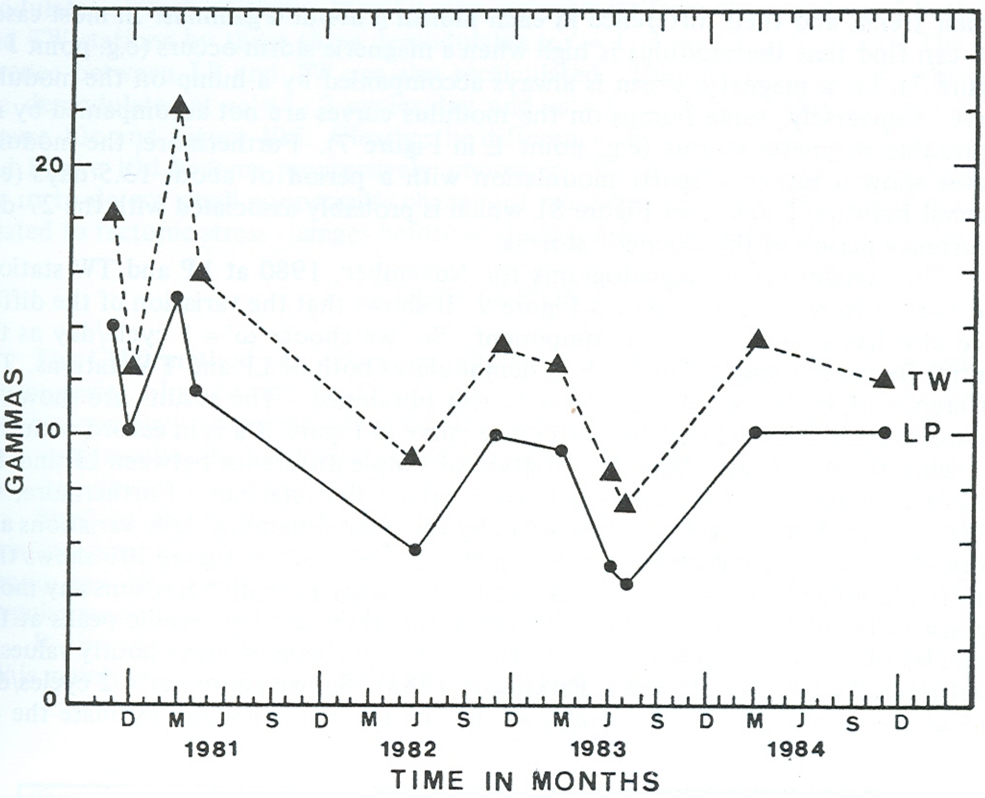

Figure 8. The monthly mean of modulus.

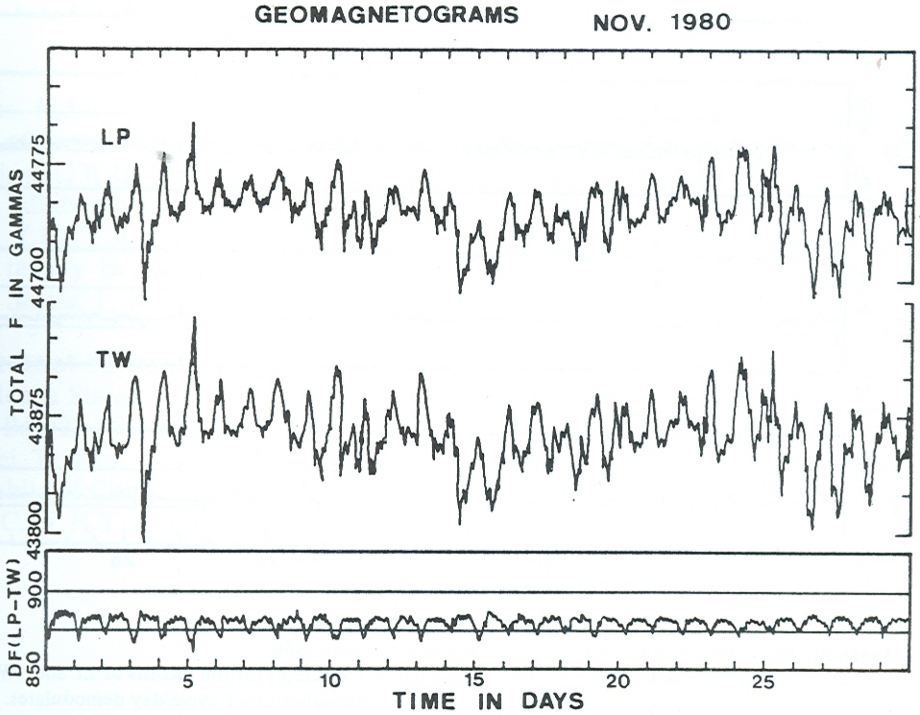

Figure 9. Synchronized magnetograms at LP and TW for November, 1980 and their difference (DF).

similar trend, and their difference in each month is about 4 gammas. In most cases, one can find that the modulus is high when a magnetic storm occurs (e.g. point J in Figure 7), i.e. a magnetic storm is always accompanied by a hump on the modulus curve. Conversely, some humps on the modulus curves are not accompanied by recognizable magnetic storms (e.g. point L in Figure 7). Furthermore, the modulus curves show a rather irregular modulation with a period of about 13.5 days (e.g. interval between J and L in Figure 8), which is probably associated with the 27-day recurrence period of the magnetic storms.

The synchronized magnetograms for November, 1980 at LP and TW stations and their difference are shown in Figure 9. It shows that the variation of the difference also has a daily recurrent component. So, we choose $\omega' = 1$ cycle/day as the central frequency and calculate their demodulates both of LP and TW stations. The difference of these two demodulates is also obtainned. The results are shown in Figure 10a. It is found that the difference curve in Figure 10a is in accord with that in Figure 9. It indicates that the variation of simple difference between LP and TW is subjected to the daily variation between these two stations. Furthermore, we subtract from both of the two raw series by the demodulates of daily variations and recalculate the simple difference between them. The result in Figure 10b shows that the simple difference curve does not exhibit the daily periodic variations any more. Occasionally, in addition to the daily variations, there are two visible peaks at frequencies of 2 and 3 cycles per day in the power spectrum of mean hourly values of total intensity (Yeh, et al., 1981; Parkinson, 1983). So, we choose $\omega' = 2$ cycles/day and $\omega' = 3$ cycles/day, respectively, as the central frequency and estimate the de-

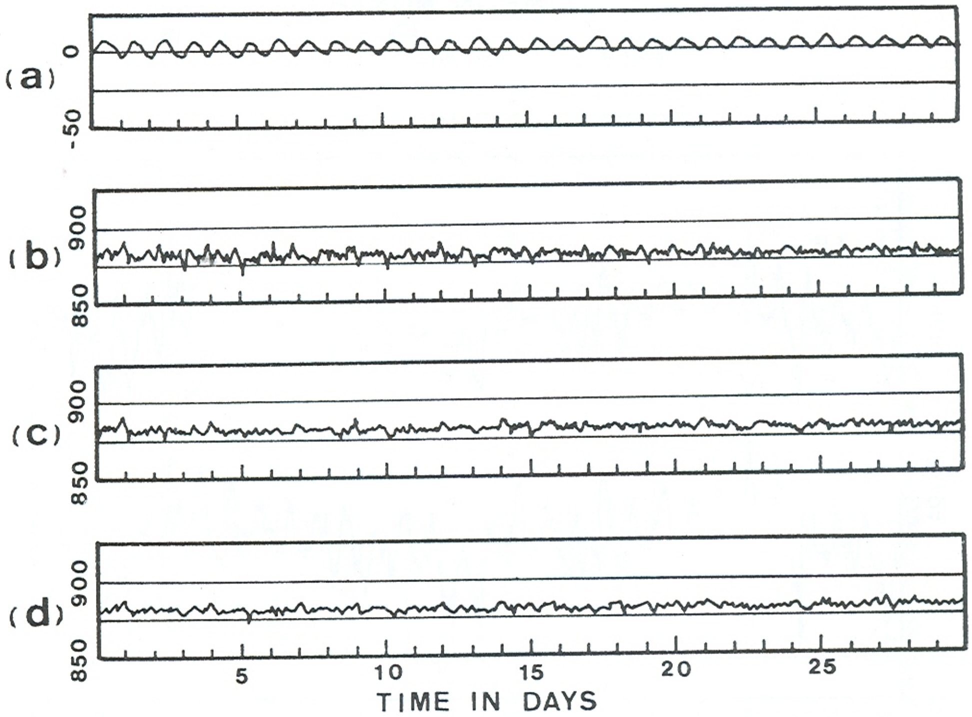

Figure 10. Difference data between LP and TW for November 1980.
(a) difference between the $\omega' = 1$ cycle/day demodulates for the records of LP and TW.
(b) difference between the records after the removal of $\omega' = 1$ cycle/day demodulates.
(c) same as (b) except that $\omega' = 1$ and 2 cycle/day demodulates have been removed.
(d) same as (b) except that $\omega' = 1$, 2 and 3 cycle/day demodulates have been removed.

modulates of each of them. Step by step, we subtract from the raw data of both LP and TW stations by these three demodulates ($\omega' = 1$, 2 and 3 cycles/day). The difference between LP and TW are also recalculated. The differences after removing the demodulates at $\omega'=1$, 2 cycles/day and $\omega'=1$, 2, 3 cycles/day are shown in Figure 10c and Figure 10d. Clearly, the difference curves in Figure 10b, Figure 10c and Figure 10d become progressively smoother. These results would make it easier for us to detect small nonperiodic changes of magnetic total intensity which may be related to tectonic stress changes before or during major earthquakes.

CONCLUSION

The CMD method can effectively filter out a unitary frequency component of a time series. In addition, the CMD method can also give the moduius and phase of any frequency component demodulate. It is difficult for other techniques to yield such results. In particular, it is good to use the CMD method when both modulus and phase change with time.

By using the CMD method, we find that the average of the moduli of daily variation demodulates of 11 magnetic total intensity records at LP and TW is 9.6 gamma and 13.2 gamma, respectively. Additionally, the modulus shows a rather irregular modulation with a period of about 13.5 days.

It also can be concluded that the variation of the difference between LP and TW is mainly subjected to the daily variation between these two stations.

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台灣地磁全磁力資料之複數解調研究

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摘 要

複數解調方法（CMD），主要是利用傅力葉轉換（Fourier Transform）中的頻率平移（frequency shifting）技巧。它可以用來濾取時間系列中的單一頻率分量，更可以用來查驗時間系列中某一頻率分量之振幅（amplitude）及相位（phase）隨時間變化情形。檢視崙坪（LP）及曾文（TW）兩站的地磁全磁力波譜，可以發現其中很明顯的存在一個24小時的週期分量。而這兩站記錄的簡易差值（Simple difference）亦顯示出類似的週期性變化現象。我們使用複數解調方法來分析這兩站的地磁資料，以求進一步瞭解這個週期現象的特性。

我們選取1周／天當做頻率分量，計算了11組不同月份的小時平均磁力值的解調函數（demodulates），並且求出與此解調函數對應的模數（modulus）及相位（phase）。而這11組模數的平均值在崙坪及曾文分別為9.6伽傌及13.2伽傌。